Experimental demonstration of photonic quantum Fredkin gate and its applications

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Quantum Fredkin gate represents one of the fundamental quantum gates with many potential applications in quantum information processing. Here we report on experimental realization of an optical quantum Fredkin gate for qubits encoded into states of single photons. We utilize this gate for implementation of several important quantum information protocols such as optimal quantum cloning [1, 2], optimal purification of single qubits [3], or direct nondestructive measurements of purity and overlap of quantum states [4, 5].

As shown in Fig. 1, our Fredkin gate is realized as a sequence of a two-qubit quantum CNOT gate, a three-qubit quantum Toffoli gate (a CCNOT gate), and another two-qubit CNOT gate. Combination of several linear optical quantum gates becomes possible due to our specific qubit encoding, where two qubits are encoded into polarization and path degrees of freedom of one photon, while the third qubit is encoded into polarization of a second photon. With this qubit representation, the CNOT gates can be implemented deterministically with linear optics while the quantum Toffoli gate is realized by two-photon interference on an unbalanced beam splitter [6]. The core of our setup is composed of six calcite beam displacers which form an inherently stable multipath optical interferometer. Among other tasks, the beam displacers realize a deterministic SWAP operation between the polarization and path qubits, which enables us to implement all other gates by addressing the polarization degrees of freedom with wave plates and partially polarizing beam splitters.

We have performed full tomographic reconstruction of the implemented quantum Fredkin gate. This tomographic characterization yielded a quantum gate fidelity of $F = 0.901(1)$. By preparing the control qubit of the Fredkin gate in the superposition state $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ and measuring this qubit at the gate output in the superposition basis $\frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$ we can perform symmetrization or anti-symmetrization of the state of the two target qubits. This is a fundamental quantum operation which has many important applications including optimal quantum cloning [1, 2], purification of single-qubit states [3], implementation of a programmable universal quantum measurement device [7], and measurement of purity and overlap of quantum states [4, 5].

We have successfully tested all these applications with our quantum Fredkin gate and investigated some of their peculiar properties, such as the non-destructive character of the purity measurement. This latter application requires two copies of the quantum state ρ whose purity should be measured. The target qubits are prepared in the state $\rho \otimes \rho$ and the purity is estimated from the measurements on the control qubit. Remarkably, the reduced density matrices of the output target qubits remain unperturbed at the output of the gate. However, the purity measurement introduces correlations between the two copies of the state which prevents subsequent extraction of any further information on the state purity by the same procedure.

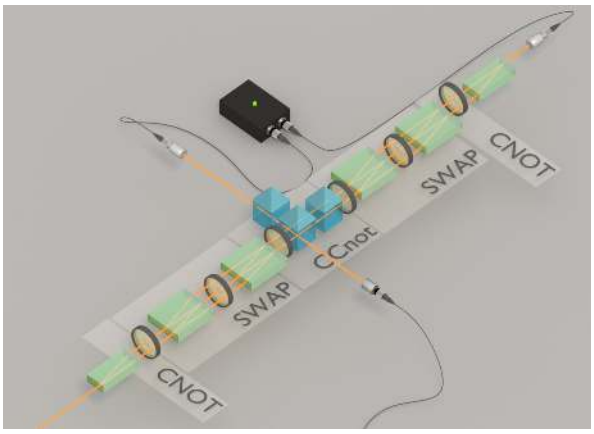

FIG. 1: Schematic experimental setup of the linear optical quantum Fredkin gate. Calcite beam displacers are indicated by light green color and the partially polarizing beam splitters have a blue color.

Our comprehensive characterization of the performance of quantum Fredkin gate clearly confirms its versatility and usefulness in various areas of quantum information processing.

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