Ultimate precision limit for noisy frequency estimation

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Quantum metrology protocols allow to surpass the precision limits typical to classical statistics. Given an increasing number N of probes independently prepared to sense a parameter of interest, the central limit theorem constraints the estimation error to decrease at most as $1/N$, while the use of initially entangled probes enables in principle to reach the $1/N^2$ scaling, the so-called Heisenberg limit [1]. The interaction of the probes with the environment is known to limit considerably such an improvement, possibly reducing it to a constant factor over the classical scenario [2,3]. Here [4], we present a novel attainable limit to the precision of the frequency estimation in the presence of noise, which holds for a wide class of qubit open-system dynamics and includes several physical scenarios of practical interest. Exploiting the time inhomogeneous nature of the noisy evolution, we show that the classical limit can be actually overcome by means of initially entangled states, even if the Heisenberg limit is not within reach. The optimal strategy to minimize the estimation error involves measurements on shorter and shorter times, with the increasing of N, and makes use of the deviations from the exponential decay, which are typical of the quantum Zeno regime. We demonstrate that the ultimate attainable precision is fixed by the short-time expansion of the effective noise parameters, irrespective of any possible subsequent memory (non-Markovian) effect. The metrological limit we derive relies on recently introduced powerful techniques [5,6] to evaluate the quantum Fisher information of generic quantum states, and on general properties of the open-system dynamics. As a consequence, our results can be applied to very different real-world settings, without the need for a detailed characterization of the environmental features or the interaction mechanisms.

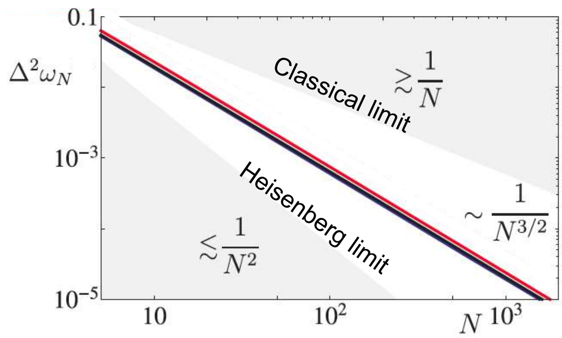

FIG. 1: The interaction with an environment with a finite correlation time allows for noisy metrological limits that surpass the standard quantum limit imposed by the central limit theorem. We demonstrate the existence of a fundamental (Zeno) limit (in black) to the best attainable precision in noisy frequency estimation and its attainability (in red) using quantum probes initially prepared in an entangled state. While the asymptotic Zeno-resolved precision is above the Heisenberg unitary limit, no further improvements are feasible by exploiting non Markovian effects and therefore this bound provides a fundamental limit to the resolution for a broad class of system-environment interactions.

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