Ultimate precision bounds for the estimation and discrimination of quantum channels [1]

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Quantum metrology deals with the optimal estimation of physical parameters encoded in quantum states or transformations. Its applications are many, from enhancing gravitational wave detectors, to improving frequency standards, clock synchronization and optical resolution. Understanding the ultimate precision limits of quantum metrology is therefore of paramount importance. However, it is also challenging, because the most general strategies for quantum parameter estimation exploit adaptive, i.e., feedback-assisted, quantum operations involving an arbitrary number of ancillas [2, 3].

Our goal is to estimate the ultimate precision in the estimation of θ, as given by the quantum Cramér-Rao bound

$$\sigma_\theta^2 \geq \frac{1}{F_\theta(\rho_{AB}^n)},$$

where F_θ is the quantum Fisher information and ρ_{AB}^n is the final state after n iterations, see Fig. 1. To solve this problem we borrow the powerful tool of *teleportation stretching* from the field of quantum communication [4]: if the channel $\mathcal{E}_\theta$ has a suitable symmetry, its action on any input ρ can be simulated by local operations and classical communication (LOCC), see Fig. 2. In this way, the action of the quantum channel on generic inputs is naturally incorporated in the adaptive estimation protocol, allowing us to derive an upper bound on the quantum Fisher information and thus on the ultimate precision for the estimation of the parameter θ. This simulation is possible for channels that are covariant under the action of the unitary transformations involved in the teleportation protocol [5]: examples are the depolarizing and erasure channels, and the Gaussian channels in bosonic systems.

Together with the upper bound we find a matching lower bound obtaining a remarkably simple expression for the ultimate quantum Fisher information:

$$F_\theta(\rho_{AB}^n) = nF_\theta(\rho_{\mathcal{E}_\theta}),$$

where $\rho_{\mathcal{E}_\theta}$ is the Choi-Jamiołkowski state associated to $\mathcal{E}_\theta$. This finding shows that the adaptive estimation of noise in a teleportation-covariant channel cannot beat the standard quantum limit. Our no-go theorem also establishes that this limit is achievable by using entanglement without adaptiveness.

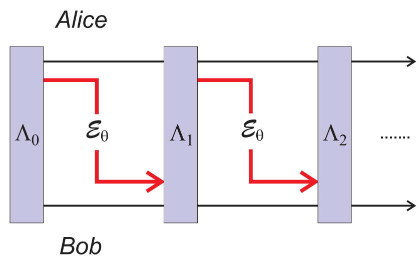

FIG. 1: Schematics for the most general adaptive estimation protocol. First Alice and Bob prepare an initial state by applying a quantum map Λ_0, then Alice uses part of this state to probe the box, while Bob gets the corresponding output. Then they apply a collective quantum operation Λ_1, Alice prepares a new input state, and so on and so forth for n concatenation of this adaptive routine. The state of Alice and Bob obtained in this way, denoted as ρ_{AB}^n, it is finally measured to estimate θ.

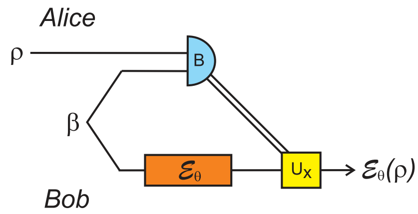

FIG. 2: Teleportation allows us to simulate the quantum channel $\mathcal{E}_\theta$ with an entangled resource (the Choi-Jamiołkowski state of $\mathcal{E}_\theta$) and LOCC, provided it has the required symmetry.

As an application, we set the ultimate adaptive limit for estimating thermal noise in Gaussian channels, which has implications for continuous-variable quantum key distribution and, more generally, for measurements of temperature in quasi-monochromatic bosonic baths. Because our methodology applies to any functional of quantum states which is monotonic under completely- positive trace-preserving maps, we are able to simplify other types of adaptive protocols, including those for quantum hypothesis testing. Similarly, we find that the ultimate error probability for discriminating two teleportation-covariant channels is reached without adaptiveness and determined by their Choi-Jamiołkowski states.

Our work not only shows that teleportation is a primitive for quantum metrology but also provides remarkably simple and practical results. Setting the ultimate precision limits of noise estimation and discrimination has broad implications, e.g., in quantum tomography, imaging, sensing and even for testing quantum field theories in non-inertial frames.

[1] S. Pirandola, C. Lupo, Phys. Rev. Lett. **118**, 100502 (2017)
[2] R. Demkowicz-Dobrzański, L. Maccone, Phys. Rev. Lett. **113**, 250801 (2014).
[3] M. Hayashi, Commun. Math. Phys. **304**, 689 (2011).
[4] Pirandola et al., arXiv: 1510.08863 (2015).
[5] C. H. Bennett et al., Phys. Rev. Lett. **70**, 1895 (1993).